AP Calculus Summer Packet

Writing the Equation Of A Line

Notes

Example: Find the equation of a line that passes through $(-1, 2)$ and $(5, 7)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-2}{5+1} = \frac{5}{6}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{5}{6}(x - -1)$$

$$y - 2 = \frac{5}{6}x + \frac{5}{6} \quad \text{or} \quad y = \frac{5}{6}x + \frac{17}{6}$$

✓ **Things to remember:** Slope formula, point-slope form, slope-intercept form, parallel lines have equal slope, perpendicular lines have slopes that are opposite reciprocals, vertical lines have undefined slope, horizontal lines have a slope of zero

Practice Problems

Write the equation of the line described below.

1) Passes through the point $(2, -1)$ and has slope $-\frac{1}{3}$.

2) Passes through the point $(4, -3)$ and is perpendicular to $3x + 2y = 4$.

3) Passes through $(-1, -2)$ and is parallel to $y = \frac{3}{5}x - 1$.

4) Passes through the points $(1, -2)$ and $(2, 1)$.

5) Passes through the points $(2, 3)$ and $(-1, 3)$.

6) Write an equation for a line that passes through $(2, 3)$ and is (a) horizontal and (b) vertical.

Asymptotes

Notes

Example: Find the vertical and horizontal asymptotes of $y = \frac{5x^2-4x-1}{x-1}$

$$\frac{(5x+1)\cancel{(x-1)}}{\cancel{x-1}}$$

No vertical asymptotes

$$\lim_{x\to\infty} 5x+1 = \infty \qquad \lim_{x\to-\infty} 5x+1 = -\infty$$

No horizontal asymptotes

✓ **Things to remember:** End behavior is the same thing as horizontal asymptotes! Vertical asymptotes occur at non-removable discontinuities!

Practice Problems

Identify any vertical or horizontal asymptotes.

7) $y = \frac{1}{x-1}$

8) $y = \frac{2x^2}{3x^3-4x+1}$

9) $y = \frac{x^2}{x^2-1}$

10) $y = \frac{x-4}{-4x+16}$

11) $y = \frac{3x^2}{2x^2-3x+3}$

12) $y = \frac{x^3-9x}{-4x^3+4x^2+24x}$

Factoring

Notes

Example: Factor $9x^2 + 3x - 3xy - y$ completely.

$9x^2 + 3x - 3xy - y$

$3x(3x+1) - y(3x+1)$

$(3x+1)(3x-y)$

$(3x+1)(3x-y)$

✓ **Things to remember:** First thing you look for is a GCF! If you have four terms the best way to approach factoring is by grouping! Don't forget about difference of squares and sum and difference of cubes!

Practice Problems

Factor Completely.

13) $64x^6 - 1$

14) $42x^4 + 35x^2 - 28$

15) $6x^3 - 17x^2 + 5x$

Simplify.

16) $\frac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$

Exponential And Radical Form

Notes

Example: Convert $\frac{\sqrt[7]{a^3}}{\sqrt[3]{a}}$ from radical form to exponential form.

$$\frac{a^{3/7}}{a^{1/3}} = a^{\frac{3}{7}-\frac{1}{3}}$$
$$= a^{\frac{9}{21}-\frac{7}{21}}$$
$$= a^{\frac{2}{21}}$$

✓ **Things to remember:** Rationalizing involves multiplying by the conjugate.

Practice Problems

Rationalize the numerator or denominator.

17) $\frac{x}{1-\sqrt{x-2}}$

18) $\frac{\sqrt{x+1}+1}{x}$

Simplify the exponential expression.

19) $\frac{(x^{-3}y^2)^2}{(x^4y^3)^3}$

20) $\left(\frac{a^3b^{-2}}{c^4}\right)^2\left(\frac{a^4c^{-2}}{b^3}\right)^{-1}$

Convert from radical form to exponential form or vice versa.

21) $\sqrt[5]{x^4}$

22) $(81m^6)^{\frac{1}{2}}$

23) $9^{\frac{1}{2}}$

24) $\frac{\sqrt[7]{x^9}}{\sqrt[5]{x^6}}$

Parent Functions And Transformations

Notes

Example: Describe how $f(x) = x^3$ was transformed into $g(x) = (-x)^3 + 3$.

$F(x)=x^3$

parent function

$g(x)= (-x)^3 + 3$ shifted up 3

reflected over y

Practice Problems

Sketch the graph of the given parent function.

25) $y = \sqrt{x}$

26) $y = x^3$

27) $y = \ln x$

28) $y = |x|$

29) $y = e^x$

30) $y = \sqrt[3]{x}$

31) $y = x^2$

32) $y = \frac{1}{x}$

Describe how $f(x)$ was transformed into $g(x)$.

33) $f(x) = x^2$ $\quad$ $g(x) = (x+2)^2 - 3$

34) $f(x) = |x|$ $\quad$ $g(x) = |x-5| + 2$

Even And Odd Functions

Notes

Example: Determine if $y = x^3 + 3x$ is even, odd, neither, or both.

$y = x^3 + 3x$
$y = (-x)^3 + 3(-x)$
$y = -x^3 - 3x$
Not Even!

$y = x^3 + 3x$
$-y = (-x)^3 + 3(-x)$
$-y = -x^3 - 3x$
$y = x^3 + 3x$
Odd!

✓ **Things to remember:** To test for symmetry with the y-axis (even) replace x with $-x$ and see if you get what you started with! To test for symmetry with the origin (odd) replace x and y with $-x$ and $-y$ respectively and see if you get what you started with!

Practice Problems

Determine if the function is even, odd, neither, or both.

35) $y = x^4 - 6x^2 + 3$

36) $x^2 + y^2 = 64$

37) $y = \frac{x^3 - x}{x^2}$

38) $y = \frac{1}{x^2 - 1}$

39) $y = x^4 + x^2$

Logarithmic and Exponential Functions

Notes

Example: Simplify $\log_{125}\frac{1}{5}$

$$\log_{125}\frac{1}{5}$$

$$125^x = \frac{1}{5}$$

$$5^{3x} = 5^{-1}$$

$$3x = -1$$

$$x = -1/3$$

✓ **Things to remember:** Logarithmic and exponential equations are inverses of one another- you use them to solve one another!

Practice Problems

Simplify the following expressions.

40) $\log_4\frac{1}{16}$

41) $\ln e$

42) $3\log_3 3 - \frac{3}{4}\log_3 81 + \frac{1}{3}\log_3\frac{1}{27}$

43) $\ln 1$

44) $\log_9 27$

45) $\ln\frac{1}{e^3}$

46) $\log_w w^{45}$

Solve for x. Round to three decimal places.

47) $2^x = 5$

48) $\log x^2 - \log 100 = \log 1$

49) $e^x - 4 = 0$

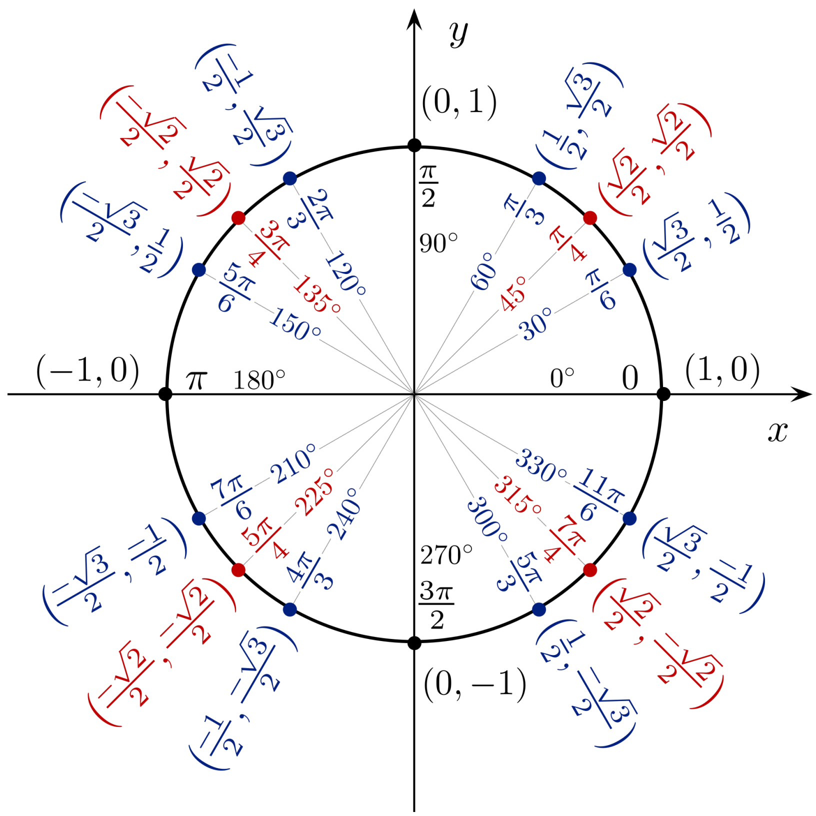
50) $3^{x+1} = 15$

51) $\log_6(x+3) + \log_6(x+4) = 1$

the Unit Circle

Notes

You need to have this memorized!



✓ **Things to remember:** Reference angles, entire unit circle, and using unit circle to find inverse trigonometric functions!

Practice Problems

Find the following.

52) $\sec\frac{-\pi}{6}$

53) $\cot 8\pi$

54) $\tan\frac{9\pi}{4}$

55) $\tan\frac{5\pi}{2}$

56) $\cos\frac{11\pi}{3}$

57) $\csc\frac{-5\pi}{6}$

58) $\sin\frac{11\pi}{4}$

59) $\sin\frac{7\pi}{3}$

60) $\arcsin 1$

61) $\arccos \frac{\sqrt{3}}{2}$

62) $\cos^{-1}\left(\cos \frac{1}{2}\right)$

63) $\sin^{-1} \frac{-\sqrt{2}}{2}$

64) $\sin\left(\arccos \frac{\sqrt{3}}{2}\right)$

Convert from radians to degrees or degrees to radians.

65) $\frac{\pi}{3}$

66) 45°

67) -9°

Solve the following equations on the given interval.

68) $\cos^2 x = \cos x + 2, \;\; 0 \leq x \leq 2\pi$

69) $2\sin(2x) = \sqrt{3}, \;\; 0 \leq x \leq 2\pi$

70) $4\cos^2 x = 1, \;\; 0 \leq x \leq 2\pi$

Trigonometric Identities

Notes

You need to have this memorized!

Reciprocal Identities	$\sin\theta = \frac{1}{\csc\theta}$ $\cos\theta = \frac{1}{\sec\theta}$ $\tan\theta = \frac{1}{\cot\theta}$	$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$
Quotient Identities	$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$
Pythagorean Identities	$\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $\cot^2 x + 1 = \csc^2 x$	

✓ **Things to remember:** Be able to use these identities to simplify trigonometric expressions! It is important that you memorize these!

Practice Problems

Simplify the following expressions.

71) $\frac{(\tan^2 x \cdot \csc^2 x) - 1}{\csc x \cdot \tan^2 x \cdot \sin x}$

72) $\sec^2 x - \tan^2 x$

73) $1 - \cos^2 x$

74) $(\csc x - \tan x)\cos x$

Limits And Continuity

Notes

Example: Use limits to show that $f(x) = \frac{x^2+3x+2}{x+1}$ is not continuous at $x = -1$.

$f(x) = \frac{x^2+3x+2}{x+1}$

x	-1.02	-1.01	-1	-.99	-.98
f(x)	.98	.99	und	1.01	1.02

$\lim_{x\to -1^-} f(x) = \underline{1}$ $\lim_{x\to -1^+} f(x) = \underline{1}$ $\lim_{x\to -1} f(x) = \underline{1}$ $f(-1) = \underline{\text{und}}$

$\lim_{x\to -1} f(x) \neq f(-1)$. So, $f(x)$ is not continuous @ $x=-1$

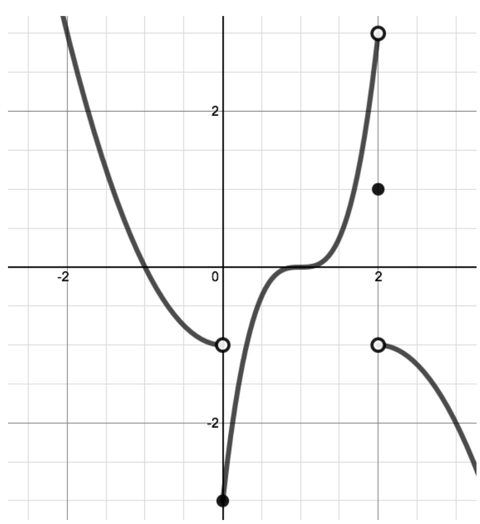
✓ **Things to remember:** The continuity test and the three types of discontinuities!

Practice Problems

Evaluate the following limits using a table.

75) $\lim_{z\to 0^-} \frac{\sqrt{z+9}-3}{z} =$ 76) $\lim_{z\to 0^+} \frac{\sqrt{z+9}-3}{z} =$ 77) $\lim_{z\to 0} \frac{\sqrt{z+9}-3}{z} =$

Evaluate the following limits using the graph provided below.



78) $\lim\limits_{x \to 0^-} f(x) =$

79) $\lim\limits_{x \to 0^+} f(x) =$

80) $\lim\limits_{x \to 0} f(x) =$

81) $f(0) =$

82) Is the graph of $f(x)$ continuous at $x = 0$? If not continuous, what type of discontinuity occurs at $x = 0$.

83) $\lim\limits_{x \to 1^-} f(x) =$

84) $\lim\limits_{x \to 1^+} f(x) =$

85) $\lim\limits_{x \to 1} f(x) =$

86) $f(1) =$

87) Is the graph of $f(x)$ continuous at $x = 1$? If not continuous, what type of discontinuity occurs at $x = 1$.